APPLICATON OF MORSE DECOMPOSITION ON VECTOR AND TENSOR FIELDS

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ABSTRACT

Morse decomposition is an effective technique for analyzing the topological structure vector fields in a reliable manner. Using Morse decomposition to form a Morse connection graph (MCG) yields results that are not susceptible to small perturbations in the vector field like noise, the underlying mesh structure, or the type of interpolation scheme used to find the edges between triangles. This is in contrast to the techniques used to find individual trajectories like fixed points, separatrices and periodic orbits, which can be quite different even when the vector field is changed slightly. Because of the stability gained from Morse decomposition, results from an MCG may be examined with much greater assurance that the underlying structure of the field has been correctly identified. This has been shown to work on vector fields, and we would like to suggest that the same techniques could be applied to tensor fields even higher degree N-RoSy fields.

INTRODUCTION

When analyzing the topology of vector fields, extracting individual trajectories such as fixed points, separatrices and periodic orbits is unreliable. Those trajectories may be impacted by the way the mesh is constructed or even by noise. To obtain a higher degree of assurance in the results, Morse decomposition may be used to construct a Morse connection graph (MCG), which reveals areas of topological interest on a broader level than extracting individual trajectories. These techniques are explored by Chen et al. [1] and will be discussed at a greater length in the background.

The goal of this project was to take the techniques used in [1] and apply them to 4-RoSy fields. However, due to time constraints and unexpected results, we scaled back the project to examine the results of Morse decomposition on tensor fields.

BACKGROUND

Morse Decomposition

Topologically speaking, there are three types of features that are analyzed, and they are fixed points, separatrices and periodic orbits. Furthermore, there are three main types of fixed points: sources, sinks and saddles. Examples of each of the basic features can be seen in Figure 1. When there exist a source and a sink in close proximity, a dipole is formed (Figure 2), and if

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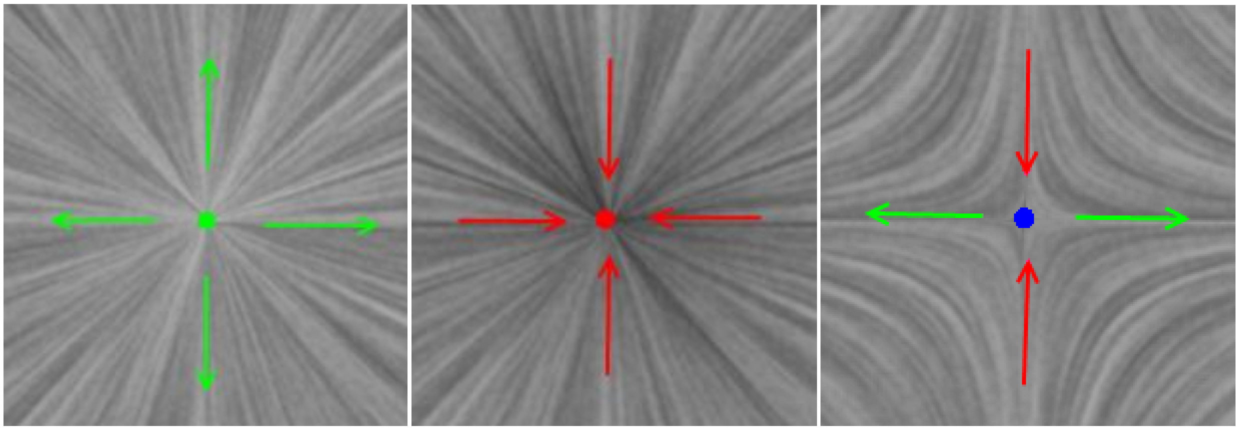


Figure 1: A source (left) is a fixed point where all flow directions are going out from the point and none are coming in to the point. Similarly, a sink (middle) is a fixed point where all of the flow directions are coming in to the point and none are going out. A simple saddle (right) is neither a source nor a sink. With a saddle, half of the flow direction is coming into the point and half is going out of the point.

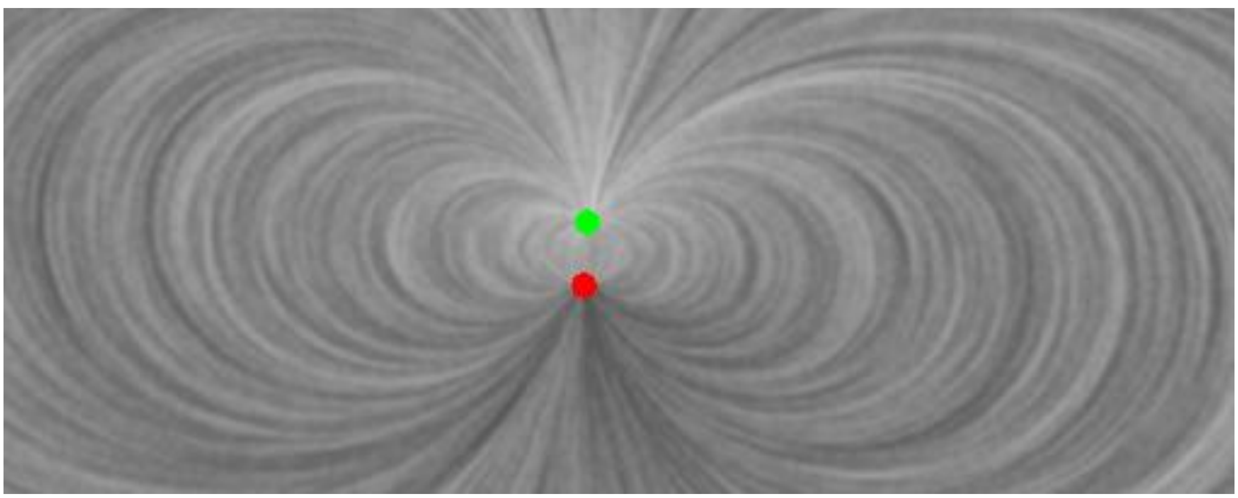


Figure 2: A dipole (left) is formed when a source and a sink are located near one another. The source is denoted by the green point and the sink is shown as the red point.

the field is reflected, the dipole becomes a monkey saddle (Figure 3), which is the simplest type of saddle that can be found in tensor fields.

To find the features of the field, Morse decomposition techniques are used to build a Morse connection graph. Essentially, every triangle is treated as a node in the graph. The vector at that particular triangle indicates the flow, and the triangles are connected in a directed graph called the entity connection graph (ECG) based on the flow vectors. Morse decomposition breaks down the ECG into its strongly connected components. These components become the new nodes and edges are consolidated so that there is only one unique edge originating from a particular component and ending at another specific component. The process of building the graph is detailed in [1].

There are several different techniques that can be used to build MCGs. The first to be explored was the geometry-based method; however, its results are generally coarse and may group several elements into one set. To refine this model, the Chen et al. introduced the τ-based method [1]. This method traces the flow for a certain time step, τ. Because of the introduction of the time step, the τ-based method results in smaller, more refined invariant sets, thus revealing more information about the topological structure of the vector field.

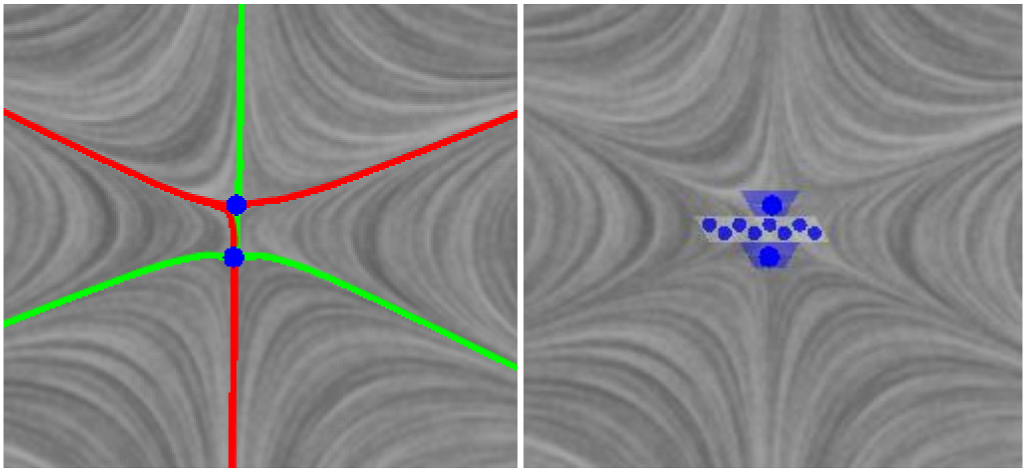


Figure 3: After reflecting the field that contained the dipole in Figure 2, the dipole becomes a monkey saddle. The left image shows the monkey saddle actually consists of two saddle points, and they are drawn with their separatrices. The image to the right shows the monkey saddle after geometry-based Morse decomposition has been run on the field. The two blue regions represent a Morse set and indicate that at least one fixed point (in this case, a saddle) is located within those triangles. The dotted region indicates that the two sets are connected. In the image to the left, the two points appear to be connected; however, methods for extracting individual trajectories are susceptible to small perturbations in the vector field so that connection may not actually exist. The Morse decomposition provides a higher degree of confidence that there is a connection between the two points.

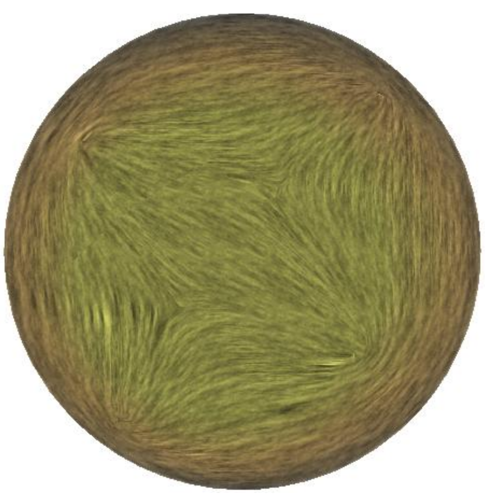
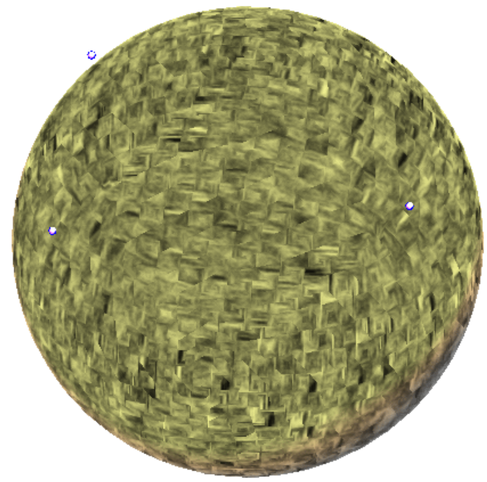
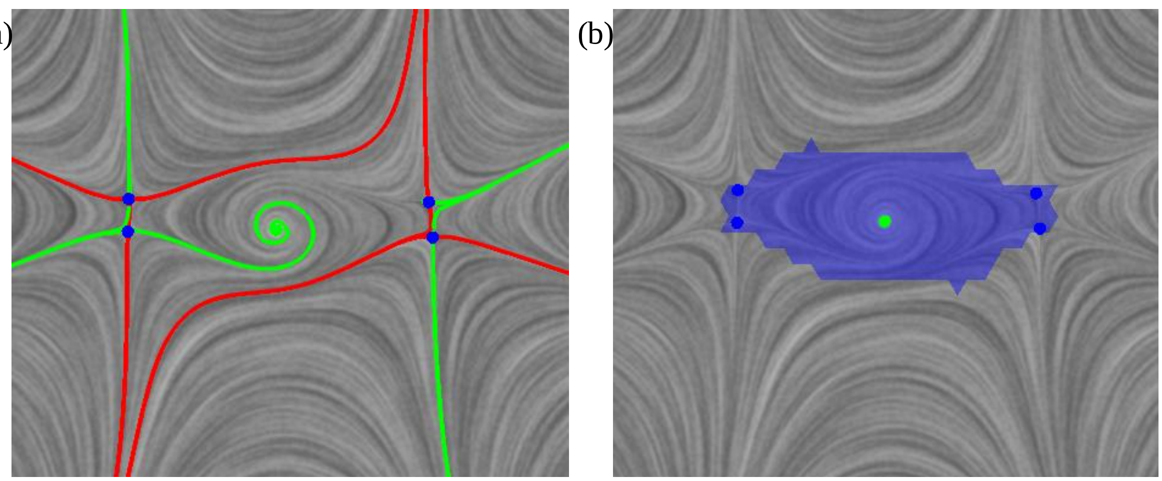


Figure 4: (left) A cut graph of the covering space of the primary sphere model used in this project. This field was obtained by creating a tensor field with two trisectors on the sphere model. The goal was to use Morse decomposition to see if the two monkey saddles, which can be seen here, actually connect. However, the monkey saddles were not detected by the program so the results of Morse decomposition may be incorrect.

Figure 5: (Right) shows a cube covering space over the sphere model resulting from a 4-RoSy field. The fixed points are marked with white spheres. The top left fixed point is drawn above the model instead of being placed on the surface of the model. The other two fixed points shown here are also drawn above the surface of the model. Their coordinates are incorrect and so the results of Morse decomposition with this program also have a high probability of being incorrect.



(a) (b)



(c)

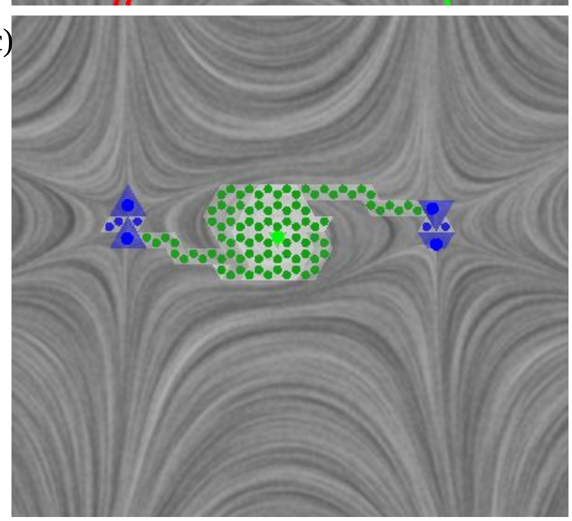


Figure 6: Note that this series of images were generated on a vector field. (a) Two monkey saddles in close proximity, drawn with separatrices. (b) The same two monkey saddles after geometry-based Morse decomposition was performed on the field. This demonstrates how course the decomposition is because both saddles are contained in the same Morse set. (c) The same field after τ-based Morse decomposition is performed (τ = 40). This decomposition shows each simple saddle in its own Morse set. The blue dotted region shows the connection between each of the simple saddles and the green dotted region shows that the two monkey saddles are actually connected.

N-RoSy Fields

We would like to find a way to apply techniques from Morse decomposition of vector fields to N-RoSy fields. N-RoSy fields give a way to describe a model in such a way that the field is invariant under integer rotations of $\frac{2\pi}{N}$ [2]. By utilizing Morse decomposition on N-RoSy fields, it would be possible to learn whether or not certain features were connected and this could lead to better fields.

RESULTS

For this project, we generated several different covering spaces for a 2-RoSy field—a tensor field—and a 4-RoSy field on a sphere model. In the covering space, the areas in the field where there are trisectors become monkey saddles (Figure 3). These should be detected as a type of fixed point during the Morse decomposition. We modified the visualization section of the program developed for [1] and used it to perform Morse decomposition on our models. Interestingly, it was unable to locate the fixed points on any of the covering spaces generated from 2-RoSy fields (Figure 4) and placed the fixed points in unexpected locations on the covering space generated from the 4-RoSy field (Figure 5).

The most surprising results came from analysis of the model seen in Figure 4. The field that was generated contained two trisectors, which means that in the covering space they become two monkey saddles. We were hoping to be able to find, through the use of Morse decomposition, that the monkey saddles do connect in the covering space just as they do in a vector field as seen in Figure 6. However, since the program did not even detect the presence of the monkey saddles, the results of the Morse decomposition cannot be relied upon either.

Because of time constraints, we were unable to fully find an answer as to why the program was not finding fixed points. In spite of that, the results so far indicate that this subject should be studied more completely. It still appears that techniques used on vector fields could be applied to tensor and higher degree N-RoSy fields.

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